Assaf, A., Mokni, K., & Gil-Alana, L. A. (2023). Long Memory and Change in Persistence in the Rare Earth Market Index. *Energy RESEARCH LETTERS*, *4*(4). https://doi.org/10.46557/001c.36532

Peer-reviewed research

Long Memory and Change in Persistence in the Rare Earth Market Index

Ata Assaf[1a], Khaled Mokni[2], Luis Alberiko Gil-Alana[3]

[1] University of Balamand, Lebanon, [2] College of Business Administration, Northern Border University, Saudi Arabia., [3] Navarra Center for International Development, University of Navara, Spain

Keywords: Rare earth elements, Long memory, Persistence changes, JEL Q32 C12 C13

https://doi.org/10.46557/001c.36532

Energy RESEARCH LETTERS

Vol. 4, Issue 4, 2023

This paper investigates long memory and change in persistence of the rare earth market index. We test for the presence of long memory, complemented with tests that account for regime change or breaks. Our findings confirm the presence of long memory in the rare earth market index, with the memory parameters increasing within two-regime schemes. A break in persistence is also estimated to have occurred on September 4, 2019.

I. Introduction

The elements of rare earth components (REEs) have a wide range of atomic numbers and have been used in areas such as industrial, defense, energy, and military technology applications (Baldi et al., 2014; Wang et al., 2019). Furthermore, those elements have been rising rapidly, with China leading world production of them, and accounting for more than 90 percent of world supply and production. To retain its limited resources in this area and deal with environmental issues, China, from the beginning of 2010, started putting some restrictions and limits on the supply of REEs through different measures. China is imposing restrictions in the form of taxes, licenses, and quotas. Consequently, these assets have gained more attention through news media and internet coverage due to their critical properties and contribution to modern technology (Apergis & Apergis, 2017; Müller et al., 2016).

The prices of rare earth market products have gone through huge fluctuations in the past due to changes in the growth of demand, market, and supply constraints (Proelss et al., 2020; Schmid, 2019; Stegen, 2015). As a result, rare earth market commodities have become separate strategic commodity classes (Fernandez, 2017; Proelss et al., 2020). This might add to the inelasticity of supply and demand for rare earth elements, given that the economy might exhibit some long memory features that might translate into the persistence of rare earth markets, influencing their supply and demand.

Therefore, more attention should be paid to the time series properties of this index, especially the question of long memory behavior and breaks in this market.[1] The supply disruptions in this market caused by some events such as the trade war between US and China during 2019 and health [and other] issues arising from the COVID-19 pandemic, may require more investigation. Therefore, we attempt to study the long memory behaviour and the change in persistence in the REEs over the period of 2010 to 2021.

Research on the rare earth market include those by García et al. (2017), Hodgkinson and Smith (2021), Buchholz and Brandenburg (2018), Fernandez (2017) and Proelss et al. (2020). Baldi et al. (2014), for example, focus on the financial and economic factors related to clean energy industries and rare earth markets. Other studies explore the importance of rare earth materials for the clean energy sector (Apergis & Apergis, 2017; Baldi et al., 2014; Chen et al., 2020; Wang et al., 2019).

In this paper, we test whether long memory is true for this asset. Then, we compare different estimates of persistence, and test whether there has been a change in persistence during the sample period, especially considering the recent trade war between the US and China and the impact of the COVID-19 pandemic.

The paper has the following sections: Section 2 includes the empirical methodology, Section 3 presents the data and empirical results and Section 4 concludes the paper.

II. Empirical Methodology

To capture long-memory feature, the *ARFIMA* (p, d, q) model is proposed and defined as:

$$\Phi(L)(1-L)^d x_t = \Theta(L)\epsilon_t, \qquad (1)$$

a **Corresponding author:**
ata.assaf@balamand.edu.lb

1 For references on the study of long memory in financial markets, see Caporale et al. (2018) and Assaf et al. (2021).

when $d = 0$, x_t is a short memory process. x_t is stationary when $-0.5 < d < 0.5$, and mean-reverting and invertible for $d < 1$.

A. Long Memory Testing and Estimators

To test for long memory, Robinson (1995) proposed the spectral density function of x_t at frequency λ as:

$$f(\lambda) \sim G\lambda^{-2d} \text{ as } \lambda \to 0+, \tag{2}$$

G is a function that is slowly varying and $d \in (-0.5, 0.5)$. Then, the periodogram can be presented as:

$$I(\lambda_j) = (2\pi T)^{-1} \left| \sum_{t=1}^{T} x_t \exp(i\lambda_j t) \right|^2, \tag{3}$$

where $\lambda_j = \frac{2\pi j}{T}$, $j = 1, \ldots, \lfloor T/2 \rfloor$ are the Fourier frequencies and T is the sample size. Then, the local Whittle likelihood function is:

$$Q(G, d) = \frac{1}{m} \sum_{j=1}^{m} \left\{ \log G\lambda_j^{-2d} + \frac{I(\lambda_j)}{G\lambda_j^{-2d}} \right\}, \tag{4}$$

where m is the bandwidth parameter given by $m = \lfloor T^{\delta} \rfloor$ and $\delta = [\frac{1}{3}, \frac{4}{5}]$. In this case, the estimator of d is given by $\widehat{d} = \arg\min_d R(d)$, converging to a normal distribution as follows $\sqrt{m}(\widehat{d} - d) \xrightarrow{d} \mathcal{N}(0, \frac{1}{4})$.

B. Testing for Spurious Long Memory

Qu (2011) proposed a test to distinguish a true long memory process from a spurious one, where the null hypothesis is a stationary long-memory, and the alternative is contaminated by structural smooth trends or breaks. The test is as follows:

$$W = \sup_{r \in [\epsilon, 1]} \left(\sum_{j=1}^{m} \nu_j^2 \right)^{-\frac{1}{2}} \left| \sum_{j=1}^{\lfloor mr \rfloor} \nu_j \left(\frac{I(\lambda_j)}{G(\widehat{d})\lambda_j^{-2\widehat{d}}} - 1 \right) \right| \tag{5}$$

Then, the estimator of the $\widehat{d}$ parameter is obtained using m bandwidth instead of $\lfloor mr \rfloor$, and $m^{-\frac{1}{2}}$ replaced by $\left(\sum_{j=1}^{m} \nu_j^2 \right)^{-\frac{1}{2}}$ for size correction. Hou and Perron (2014) provide another estimator $\widehat{d} = \arg\min_{d,\theta} R(d, \theta)$ which has the following modification:

$$\begin{aligned} R(d, \theta) = \log &\left(\frac{1}{m} \sum_{j=1}^{m} \frac{I_{\ddagger}(\lambda_j)}{\lambda_j^{-2d} + \frac{\theta\lambda_j^{-2}}{T}} \right) \\ &+ \frac{1}{m} \sum_{j=1}^{m} \log\left(\lambda_j^{-2d} + \theta\lambda_j^{-2}/T \right) \end{aligned} \tag{6}$$

with m greater than $m = T^{5/9}$.

C. Testing for Change in Persistence

We follow Sibbertsen and Kruse's (2009) approach, assuming an *ARFIMA (p,d,q)* process. They proposed the following hypothesis on the change in d:

$$H_0 : d = d_0,\, \forall t \ Vs.\ H_1 : \begin{cases} d = d_1 \ for\ t = 1 \ldots\ldots [\tau T] \\ d = d_2 \ for\ t = [\tau T] + 1, \ldots\ldots T \end{cases} \tag{7}$$

Sibbertsen and Kruse's (2009) test is restricted to the case of d when considering the null hypothesis as $0 \leq d_0 < 3/2$ and as $0 \leq d_1 < 1/2$ and $\frac{1}{2} \leq d_2 < \frac{3}{2}$ under the alternative hypothesis.

Another test was suggested by Leybourne et al. (2007) as:

$$R = \frac{\inf_{\tau \in \Lambda} K^f(\tau)}{\inf_{\tau \in \Lambda} K^r(\tau)} \tag{8}$$

with the forward residuals as $K^f(\tau) = [\tau T]^{-2d_0} \sum_{t=1}^{[\tau T]} \hat{v}_{t,\tau}^2$ and the reverse residuals as $K^r(\tau) = [T - \tau T]^{-2d_0} \sum_{t=1}^{T-[\tau T]} \hat{v}_{t,\tau}^2$, where $\hat{v}_{t,\tau}$ are the residuals of a regression using the sample up to $[\tau T]$ of $x_t$on 1 for all observations.

III. Data and Empirical Results

We use daily data of the global Rare Earth/Strategic Metals Index (REMX) from November 1, 2010 to October 6, 2021 sourced from Investing.com. Results from **Table 1** indicate the acceptance of the null hypothesis of true long memory in REEs' returns and volatility. Moreover, the long memory property is also confirmed by the tests of Hou-Perron (2014) and Frederiksen et al. (2012).

We also use the ratio tests for a change in persistence proposed by Busetti and Taylor (2004), Leybourne and Taylor (2004), and Harvey et al. (2006). We then apply the modified ratio test suggested by Martins and Rodrigues (2014). **Table 2** includes the results, and provide evidence that a change in persistence has happened during the sample period. The results are also significant when considering the problem of multiple testing, suggesting a break in persistence.

When we consider the ratio test, **Table 2** adopts the suggestion proposed by Busetti and Taylor (2004). Yet, Harvey et al. (2006) provide an adjustment of the Busetti and Taylor (2004) testing approach in such a way that the critical values of the test statistic are the same under the processes of *I(0)* and *I(1)*. **Table 2** also includes Harvey et al.'s (2006) approach, and the evidence suggests a change in persistence with a break from *I(0)* to *I(1)*.

Martins and Rodrigues (2014) propose another test that can identify changes from $-1/2 < d_1 < 2$ to $-1/2 < d_2 < 2$ with $d_1 \neq d_2$. The results are reported in **Table 3**, indicating an increase in the memory parameter. **Table 3** also includes the estimation of the breakpoint under the assumption of an increase in long memory parameter. Results indicate the presence of a break in persistence on September 4, 2019, where the rare earth returns are integrated with different orders within two regimes; that is integrated with an order of 0.010 before September 4, 2019, and then the order of integration increases to 0.093 afterward.

For robustness, we allow for different breakpoint estimators, using the approach of Leybourne et al. (2007). **Table 4** includes the results, confirming those obtained by the Busetti and Taylor (2004) approach. We also estimate the long memory parameters in the two regimes based on the Geweke and Porter-Hudak (1983) approach. The results in **Table 4** locate the same breakpoint with different integration orders in both regimes, confirming an increase in memory from regime one to the second.

Table 1. Long memory tests applied to REEs index

Qu (2011) Test			Hou-Perron (2014)			Frederiksen et al. (2012)		
Returns	Absolute Returns	Log-Absolute Returns	Returns	Absolute Returns	Log-Absolute Returns	Returns	Absolute Returns	Log-Absolute Returns
0.635	0.614	0.613	0.036 (0.353)	0.426 (0.353)	0.440 (0.353)	0.248 (0.142)	0.371 (0.110)	0.467 (0.097)

Notes: The table reports the Qu (2011) test results. The test has critical values of 1.155 and 1.426 at 5% and 1%.

Table 2. Ratio Type test

	RATIO TEST				RATIO TEST-TYPE HLT					
DIRECTION	90%	95%	99%	Test Statistic	90%	95%	99%	Test Statistic90%	Test Statistic95%	Test Statistic99%
AGAINST CHANGE FROM I(0) TO I(1)	3.51	4.61	7.69	70.529	3.51	4.61	7.69	70.437	70.417	70.380
AGAINST CHANGE FROM I(1) TO I(0)	3.51	4.61	7.69	0.018	3.51	4.61	7.69	0.0184	0.0183	0.0183
AGAINST CHANGE IN UNKNOWN DIRECTION	4.63	5.88	9.24	70.529	4.63	5.88	9.24	70.418	70.400	70.364

Notes: The table reports the ratio test results based on the test proposed by Busetti and Taylor (2004), and the adjusted test statistic by Harvey et al. (2006).

Table 3. MR Test

	MR TEST				MR TEST- "STANDARD"			
DIRECTION	90%	95%	99%	Test Statistic	90%	95%	99%	Test Statistic
Against increase in memory	4.442	5.578	8.042	278.553	-1.728	-2.023	-2.589	-5.217
Against decrease in memory	4.488	5.597	8.213	0.879	-1.745	-2.070	-2.666	-0.937
Against change in unknown direction	5.496	5.566	9.238	278.553	-2.040	-2.331	-2.870	-0.937

Notes: The table reports the results for the test for a change in long memory proposed by Martins and Rodrigues (2014).

Table 4. Break Points (BP) Estimation

	BP ESTIMATION				BP ESTIMATION- LKT				BP ESTIMATION- D-ESTIMATION GPH			
	d1	d2	Breakpoint	Date	d1	d2	Breakpoint	Date	d1	d2	Breakpoint	Date
DIRECTION ***AN INCREASE***	0.010 (0.033)	0.093 (0.054)	2226	9/4/ 2019	0.010 (0.033)	0.093 (0.054)	2226	9/4/ 2019	0.035 (0.042)	0.210 (0.072)	2226	9/4/ 2019

***Notes*:** The table reports robustness analysis results that allows for different break point estimators.

IV. Conclusion

This paper investigates long memory behavior and change in persistence in the returns and volatility of the rare earth market index. Our results suggest evidence of true long memory in REEs' returns and volatility, and confirm an increase in the long memory parameter within two-regime dynamics, and having a break around September 4, 2019.

The presence of long memory in REEs suggests that the future behavior of these series cannot be predicted based on their past behavior, and traders have the potential to make abnormal profits by employing certain trading strategies. Investors may also use different persistent regimes in making their trading strategies, due to the existence of different episodes of long memory (see Cuñado et al. (2010)).

Finally, future research could be directed towards analyzing the impact of macro factors or events on REE prices, or the relationship between this market and other markets, considering multivariate long memory or fractional integration models (Assaf et al., 2021; Chen et al., 2020; Fernandez, 2017; and Reboredo & Ugolini, 2020). Forecasting models accounting for long memory can also be used to assess the risks in these markets (Letmathe et al., 2021).

References

Apergis, E., & Apergis, N. (2017). The role of rare earth prices in renewable energy consumption: The actual driver for a renewable energy world. *Energy Economics*, *62*, 33–42. https://doi.org/10.1016/j.eneco.2016.12.015

Assaf, A., Gil-Alana, L. A., & Mokni, K. (2021). True or spurious long memory in the cryptocurrency markets: Evidence from a multivariate test and other Whittle estimation methods. *Empirical Economics*, 1–28. https://doi.org/10.1007/s00181-021-02165-6

Baldi, L., Peri, M., & Vandone, D. (2014). Clean energy industries and rare earth materials: Economic and financial issues. *Energy Policy*, *66*, 53–61. https://doi.org/10.1016/j.enpol.2013.10.067

Buchholz, P., & Brandenburg, T. (2018). Demand, supply, and price trends for mineral raw materials relevant to the renewable energy transition wind energy, solar photovoltaic energy, and energy storage. *Chemie Ingenieur Technik*, *90*(1–2), 141–153. https://doi.org/10.1002/cite.201700098

Busetti, F., & Taylor, A. M. R. (2004). Tests of stationarity against a change in persistence. *Journal of Econometrics*, *123*(1), 33–66. https://doi.org/10.1016/j.jeconom.2003.10.028

Caporale, G. M., Gil-Alana, L., & Plastun, A. (2018). Persistence in the cryptocurrency market. *Research in International Business and Finance*, *46*, 141–148. https://doi.org/10.1016/j.ribaf.2018.01.002

Chen, Y., Zheng, B., & Qu, F. (2020). Modeling the nexus of crude oil, new energy and rare earth in China: An asymmetric VAR-BEKK (DCC)-GARCH approach. *Resources Policy*, *65*, 101545. https://doi.org/10.1016/j.resourpol.2019.101545

Cunado, J., Gil-Alana, L. A., & de Gracia, F. P. (2010). Mean reversion in stock market prices: New evidence based on bull and bear markets. *Research in International Business and Finance*, *24*(2), 113–122. https://doi.org/10.1016/j.ribaf.2009.10.001

Fernandez, V. (2017). Rare-earth elements market: A historical and financial perspective. *Resources Policy*, *53*, 26–45. https://doi.org/10.1016/j.resourpol.2017.05.010

Frederiksen, P., Nielsen, F. S., & Nielsen, M. Ø. (2012). Local polynomial Whittle estimation of perturbed fractional processes. *Journal of Econometrics*, *167*(2), 426–447. https://doi.org/10.1016/j.jeconom.2011.09.026

García, M. V. R., Krzemień, A., del Campo, M. Á. M., Álvarez, M. M., & Gent, M. R. (2017). Rare earth elements mining investment: It is not all about China. *Resources Policy*, *53*, 66–76. https://doi.org/10.1016/j.resourpol.2017.05.004

Geweke, J., & Porter-Hudak, S. (1983). The estimation and application of long memory time series models. *Journal of Time Series Analysis*, *4*(4), 221–238. https://doi.org/10.1111/j.1467-9892.1983.tb00371.x

Harvey, D. I., Leybourne, S. J., & Taylor, A. M. R. (2006). Modified tests for a change in persistence. *Journal of Econometrics*, *134*(2), 441–469. https://doi.org/10.1016/j.jeconom.2005.07.002

Hodgkinson, J. H., & Smith, M. H. (2021). Climate change and sustainability as drivers for the next mining and metals boom: The need for climate-smart mining and recycling. *Resources Policy*, *74*, 101205. https://doi.org/10.1016/j.resourpol.2018.05.016

Hou, J., & Perron, P. (2014). Modified local Whittle estimator for long memory processes in the presence of low frequency (and other) contaminations. *Journal of Econometrics*, *182*(2), 309–328. https://doi.org/10.1016/j.jeconom.2014.05.004

Letmathe, S., Feng, Y., & Uhde, A. (2021). Semiparametric GARCH models with long memory applied to Value at Risk and Expected Shortfall. *SSRN Electronic Journal*, *141*. https://doi.org/10.2139/ssrn.3823895

Leybourne, S., & Taylor, A. R. (2004). On tests for changes in persistence. *Economics Letters*, *84*(1), 107–115. https://doi.org/10.1016/j.econlet.2003.12.015

Leybourne, S., Taylor, R., & Kim, T.-H. (2007). CUSUM of squares-based tests for a change in persistence. *Journal of Time Series Analysis*, *28*(3), 408–433. https://doi.org/10.1111/j.1467-9892.2006.00517.x

Martins, L. F., & Rodrigues, P. M. M. (2014). Testing for persistence change in fractionally integrated models: An application to world inflation rates. *Computational Statistics & Data Analysis*, *76*, 502–522. https://doi.org/10.1016/j.csda.2012.07.021

Müller, M. A., Schweizer, D., & Seiler, V. (2016). Wealth effects of rare earth prices and China's rare earth elements policy. *Journal of Business Ethics*, *138*(4), 627–648. https://doi.org/10.1007/s10551-015-2773-3

Proelss, J., Schweizer, D., & Seiler, V. (2020). The economic importance of rare earth elements volatility forecasts. *International Review of Financial Analysis*, *71*, 101316. https://doi.org/10.1016/j.irfa.2019.01.010

Qu, Z. (2011). A test against spurious long memory. *Journal of Business & Economic Statistics*, *29*(3), 423–438. https://doi.org/10.1198/jbes.2010.09153

Reboredo, J. C., & Ugolini, A. (2020). Price spillovers between rare earth stocks and financial markets. *Resources Policy*, *66*, 101647. https://doi.org/10.1016/j.resourpol.2020.101647

Robinson, P. M. (1995). Gaussian semiparametric estimation of long range dependence. *The Annals of Statistics*, *23*(5), 1630–1661. https://doi.org/10.1214/aos/1176324317

Schmid, M. (2019). Mitigating supply risks through involvement in rare earth projects: Japan's strategies and what the US can learn. *Resources Policy*, *63*, 101457. https://doi.org/10.1016/j.resourpol.2019.101457

Sibbertsen, P., & Kruse, R. (2009). Testing for a break in persistence under long-range dependencies. *Journal of Time Series Analysis*, *30*(3), 263–285. https://doi.org/10.1111/j.1467-9892.2009.00611.x

Stegen, K. S. (2015). Heavy rare earths, permanent magnets, and renewable energies: An imminent crisis. *Energy Policy*, *79*, 1–8. https://doi.org/10.1016/j.enpol.2014.12.015

Wang, P., Chen, L.-Y., Ge, J.-P., Cai, W., & Chen, W.-Q. (2019). Incorporating critical material cycles into metal-energy nexus of China's 2050 renewable transition. *Applied Energy*, *253*, 113612. https://doi.org/10.1016/j.apenergy.2019.113612